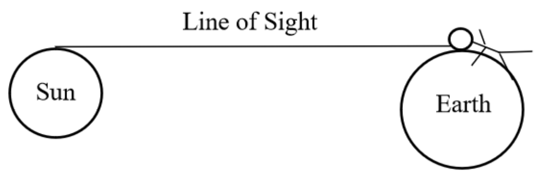
Problem 19

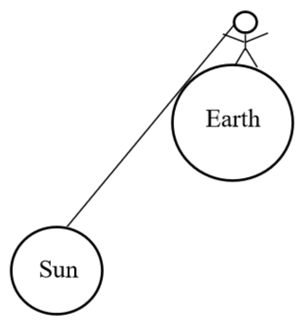
Suppose that, while lying on a beach near the equator watching the Sun set over a calm ocean, you start a stopwatch just as the top of the Sun disappears. You then stand, elevating your eyes by a height $H = 1.70$ m, and stop the watch when the top of the Sun again disappears. If the elapsed time is $t = 11.1$ s, what is the radius r of Earth?

Solution

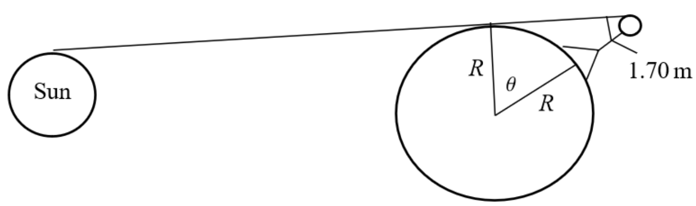
Start by drawing a picture of a person lying on the Earth with a line of sight barely touching the top of the Sun.



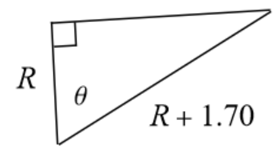
Then the person stands up to view more of the Sun.



The Earth then rotates an angle θ as the Sun disappears below the horizon.



Notice the right triangle in this picture.



Relate the angle θ with the two known sides.

$$\cos\theta = \frac{R}{R + 1.70}$$

The aim is to solve for R, the radius of the Earth (in meters). Invert both sides.

$$\frac{1}{\cos\theta} = \frac{R + 1.70}{R}$$

Split up the fraction on the right side and use the fact that $\sec\theta = 1/\cos\theta$.

$$\sec\theta = 1 + \frac{1.70}{R}$$

Isolate the term with R.

$$\sec\theta - 1 = \frac{1.70}{R}$$

Invert both sides again.

$$\frac{1}{\sec\theta - 1} = \frac{R}{1.70}$$

As a result, the formula for the radius of the Earth is

$$R = \frac{1.70}{\sec\theta - 1}.$$

To determine θ, use the fact that the Earth rotates 2π radians every 24 hours at the equator.

$$\omega = \frac{2\pi \text{ radians}}{24 \text{ hours}}$$

Convert this rotational speed to radians per second.

$$\omega = \frac{2\pi \text{ radians}}{24 \cancel{\text{hours}}} \times \frac{1 \cancel{\text{hour}}}{60 \cancel{\text{min}}} \times \frac{1 \cancel{\text{min}}}{60 \text{ s}} = \frac{\pi}{43\,200} \frac{\text{rad}}{\text{s}}$$

Multiply this rotational speed by the measured time to get θ.

$$\theta = \omega t = \frac{\pi}{43\,200} \frac{\text{rad}}{\cancel{\text{s}}} \times 11.1 \cancel{\text{s}} = \frac{37\pi}{144\,000} \text{ radians}$$

Therefore, the radius of the Earth is

$$R = \frac{1.70}{\sec\frac{37\pi}{144\,000} - 1} \approx 5.22 \times 10^6 \text{ m} = 5.22 \times 10^3 \text{ km}.$$